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On Small- x Resummations for the Evolution of Unpolarized and Polarized Non-Singlet and Singlet Structure Functions

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Abstract

A brief survey is given of recent results on the resummation of leading small- x terms for unpolarized and polarized non-singlet and singlet structure function evolution.

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ON SMALL- x RESUMMATIONS FOR THE EVOLUTION OF UNPOLARIZED AND POLARIZED NON-SINGLET AND SINGLET STRUCTURE FUNCTIONS ^a

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A brief survey is given of recent results on the resummation of leading small- x terms for unpolarized and polarized non-singlet and singlet structure function evolution.

1 Introduction

The evolution kernels of both non-singlet and singlet parton densities contain large logarithmic contributions for small fractional momenta x . In all-order resummations of these terms in the limit $x \rightarrow 0$, one naturally faces the problem of factorization and renormalization scheme dependence. Therefore these resummations have to be performed in the frame of the corresponding renormalization group equations. In the following we will discuss the resulting small- x resummations for the anomalous dimensions relevant to the different DIS processes and their quantitative consequences.

For unpolarized deep-inelastic processes the leading small- x contributions to the gluonic anomalous dimensions behave like ¹ (N is the Mellin moment)

$$\left(\frac{\alpha_s}{N-1} \right)^k \leftrightarrow \frac{1}{x} \alpha_s^k \ln^{k-1} x . \quad (1)$$

The corresponding quark anomalous dimensions, being one power down in $\ln x$, have been derived in ref. ². The leading terms of all anomalous dimensions for the non-singlet ³ and polarized singlet ⁴ evolutions are given by

$$N \left(\frac{\alpha_s}{N^2} \right)^k \leftrightarrow \alpha_s^k \ln^{2k-2} x . \quad (2)$$

The resummation of these terms can be completely derived by means of perturbative QCD. Its effect on the behaviour of the various DIS structure functions, however, is necessarily determined as well by the behaviour of the input parton

^aTalk presented by J. Blümlein.

densities at an initial scale Q_0^2 , and is therefore not predictable within perturbative QCD but has to be determined by experiment. Thus the resummation effect can only be studied via the evolution of structure functions over some range in Q^2 , which moreover probes the anomalous dimensions at all $z \geq x$ via the Mellin convolution with the parton densities.

In leading (LO) and next-to-leading order (NLO) QCD the complete anomalous dimensions are known⁵. Hence the effect of the all-order resummation of the most singular parts of the splitting functions as $x \rightarrow 0$ concerns only orders higher than α_s^2 . Due to the Mellin convolution also terms less singular as $x \rightarrow 0$ may contribute substantially at these higher orders as well. In some cases the existence of such terms is enforced by conservation laws. For the non-singlet ‘-’-evolution fermion-number conservation implies

$$\int_0^1 dz \sum_{k=1}^{\infty} \alpha_s^k P_k^-(z) = 0 . \quad (3)$$

Correspondingly, energy-momentum conservation holds for the unpolarized singlet evolution. Even in the polarized singlet case, where no conservation laws constrain the anomalous dimensions, the LO and NLO results exhibit terms which are less singular by one power in N and have about the same coefficient but with opposite sign, cf. ref.⁶. Since such contributions and further corrections are not yet known to all orders, it is reasonable to estimate their possible impact by corresponding modifications of the resummed anomalous dimensions $\Gamma(N, \alpha_s)$. Possible examples studied within refs.⁶⁻¹⁰ are:

$$\begin{aligned} \text{A : } & \Gamma(N, \alpha_s) \rightarrow \Gamma(N, \alpha_s) - \Gamma(1, \alpha_s) \\ \text{B : } & \Gamma(N, \alpha_s) \rightarrow \Gamma(N, \alpha_s)(1 - N) \\ \text{C : } & \Gamma(N, \alpha_s) \rightarrow \Gamma(N, \alpha_s)(1 - 2N + N^2) \\ \text{D : } & \Gamma(N, \alpha_s) \rightarrow \Gamma(N, \alpha_s)(1 - 2N + N^3) , \end{aligned} \quad (4)$$

where $N \rightarrow N - 1$ for the case of eq. (1). Clearly the presently known resummed terms are only sufficient for understanding the small- x evolution, if the difference of the results obtained by these prescriptions are small.

2 Resummation of dominant terms for $x \rightarrow 0$

2.1 Unpolarized non-singlet structure functions

The numerical effects due to the resummation of the $O(\alpha_s \ln^2 x)$ terms (2) have been studied in refs.^{7,8} for the structure functions $F_2^{ep} - F_2^{en}$ and $xF_3^{\nu N}(x, Q^2)$ over a wide range of x and Q^2 . The resummed terms beyond NLO lead to corrections on the level of 1% and below even at extremely small x . K -factors

of about 10 as claimed in ref. ¹¹ are not confirmed. Furthermore less singular terms can alter the resummation correction by a factor of about 3.

2.2 Unpolarized singlet structure functions

The quantitative impact of the resummation of the leading small- x terms in the gluonic ¹ and quarkonic anomalous dimensions ² has been studied for the sea quark (S) and gluon (g) distributions and the structure function F_2^{ep} in refs. ⁹ and ref. ¹⁰. The latter analysis confirms the results of the former one. Related investigations were carried out in refs. ¹².

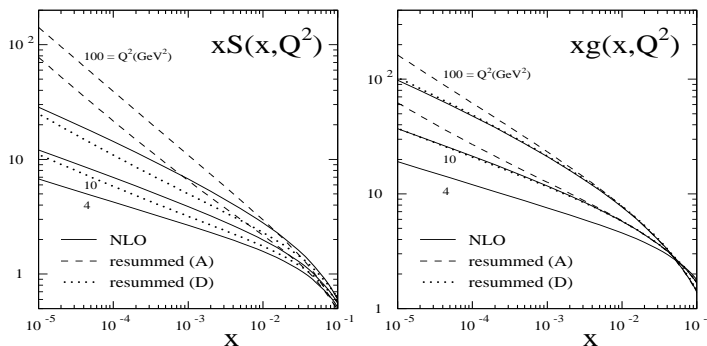


Figure 1: The evolution of the gluon and total sea densities with the resummed kernels ^{1,2} as compared to the NLO results. Two different prescriptions for implementing the momentum sum rule have been applied, cf. eq. (4). For details on initial parton densities etc see ref. ¹⁰.

In Fig. 1 the evolution of initial distribution $xS, xg \sim x^{-0.2}$ at $Q_0^2 = 4 \text{ GeV}^2$ is displayed. The effect of the resummation is very large. For the quarks and hence F_2 it is entirely dominated by the quarkonic pieces of ref. ². Two examples of additional less singular terms are shown. Their vital importance is obvious from the fact that the choice of (D) in eq. (4) leads to even an overcompensation of the enhancement due to the leading small- x terms.

2.3 Polarized non-singlet structure functions

This case was investigated in refs. ^{7,8} numerically for the structure function combination $g_1^{ep} - g_1^{en}$ for two different parametrizations of the non-perturbative initial distributions. Results on the interference structure function $g_{5,\gamma Z}^{ep}(x, Q^2)$ (cf. ref. ¹³) can be found in ref. ¹⁰. As in the unpolarized case the corrections obtained are of the order of 1% with respect to the NLO results in the kinematical ranges experimentally accessible in the foreseeable future. Huge K -factors of about 10 or larger expected for this case in ref. ¹⁴ are not present. Again less singular terms in the anomalous dimensions are only marginally suppressed.

2.4 Polarized singlet structure functions

Resummation relations for amplitudes related to the singlet anomalous dimensions of polarized DIS have recently been given⁴. Explicit analytical and numerical results for the evolution kernels beyond NLO have been derived on this basis in ref.⁶, including an all-order symmetry relation among the elements of the anomalous dimension matrix and a discussion of the supersymmetric case.

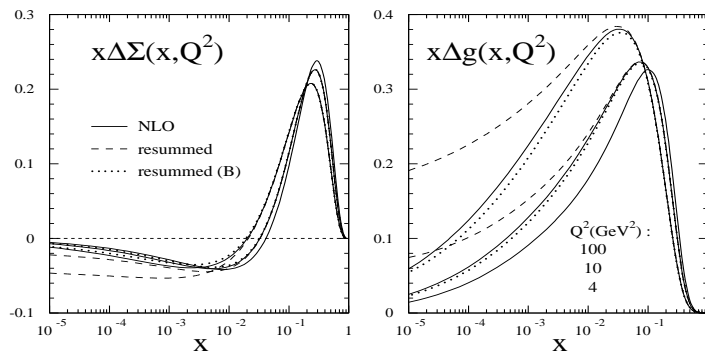


Figure 2: The resummed evolution of the polarized gluon and singlet densities as compared to the NLO results⁶. The possible impact of unknown less singular terms is illustrated by the curves (B), cf. eq. (4). The initial distributions at $Q_0^2 = 4 \text{ GeV}^2$ have been taken from ref.¹⁶.

Numerical results for the evolution of $g_1^{ep,en}(x, Q^2)$ and the parton densities have been given for different input distributions⁶. Fig. 2 shows an example. The situation is rather similar to the unpolarized case: the leading resummation effects are very large but unstable against less singular terms as $x \rightarrow 0$.

2.5 QED non-singlet radiative corrections

The resummation of the $O(\alpha \ln^2 x)$ terms may yield non-negligible contributions to QED corrections. This has been shown recently¹⁵ for the case of initial state radiation for DIS at large y . There the effect reaches around 10% of the differential Born cross section. The corresponding corrections to $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ near the Z peak are also discussed in ref.¹⁵.

3 Conclusions

The resummations of the leading small- x terms in both unpolarized and polarized, non-singlet and singlet anomalous dimensions have been investigated recently. At NLO the results agree with those found for the most singular terms as $x \rightarrow 0$ in fixed order calculations. Since the coefficient functions are known up to $O(\alpha_s^2)$ in the $\overline{\text{MS}}$ scheme, predictions for the most singular terms of three-loop anomalous dimensions have been made^{2,6-8}.

For non-singlet structure functions the corrections due to the $\alpha_s(\alpha_s \ln^2 x)^l$ contributions are about 1% or smaller in the kinematical ranges probed so far and possibly accessible at HERA including polarization^{7,8}. The non-singlet QED corrections in deep-inelastic scattering resumming the $O(\alpha \ln^2 x)$ terms can reach values of about 10% at $x \approx 10^{-4}$ and $y > 0.9$ ¹⁵.

In the singlet case very large corrections are obtained for both unpolarized and polarized parton densities and structure functions^{6,9,10}. As in the non-singlet cases possible less singular terms in higher order anomalous dimensions, however, which are in some cases required by conservation laws, are hardly suppressed against the presently resummed leading terms in the evolution: even a full compensation of the resummation effects cannot be excluded.

To draw firm conclusions on the small- x evolution of singlet structure functions also the next less singular terms have to be calculated. Since contributions even less singular than these ones may still cause relevant corrections, it appears to be indispensable to compare the corresponding results to those of future fixed order three-loop calculations.

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